Dark Energy: The Observational Challenge

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Abstract

Nearly all proposed tests for the nature of dark energy measure some combination of four fundamental observables: the Hubble parameter $H(z)$, the distance-redshift relation $d(z)$, the age-redshift relation $t(z)$, or the linear growth factor $D_1(z)$. I discuss the sensitivity of these observables to the value and redshift history of the equation of state parameter w, emphasizing where these different observables are and are not complementary. Demonstrating time-variability of w is difficult in most cases because dark energy is dynamically insignificant at high redshift. Time-variability in which dark energy tracks the matter density at high redshift and changes to a cosmological constant at low redshift is *relatively* easy to detect. However, even a sharp transition of this sort at $z_c = 1$ produces only percent-level differences in $d(z)$ or $D_1(z)$ over the redshift range $0.4 \leq z \leq 1.8$, relative to the closest constant-w model. Estimates of $D_1(z)$ or $H(z)$ at higher redshift, potentially achievable with the Lyα forest, galaxy redshift surveys, and the CMB power spectrum, can add substantial leverage on such models, given precise distance constraints at $z < 2$. The most promising routes to obtaining sub-percent precision on dark energy observables are space-based studies of Type Ia supernovae, which measure $d(z)$ directly, and of weak gravitational lensing, which is sensitive to $d(z)$, $D_1(z)$, and $H(z)$.

1 Introduction

The acceleration of the universe should come as a surprise to anyone who has ever thrown a ball into the air and watched it fall back to earth. What goes up must come down. Gravity sucks.

Repulsive gravity is possible in General Relativity, but it requires the energy density of the universe to be dominated by a substance with an exotic equation

of state, having negative pressure $p < -\rho c^2/3$. Alternatively, one can modify GR itself, so that cosmic acceleration arises even with "normal" gravitating components like pressureless matter and radiation. In this article I will adopt the language of "dark energy" solutions in which GR is preserved and a new component drives acceleration, but at present I think that a modification of the theory of gravity must be considered almost equally plausible.

Given the theoretical difficulty of explaining cosmic acceleration, it is worth emphasizing that the observational evidence for it is impressively robust. The results of the high-redshift supernova campaigns published in the late 1990s [1,2] provided the first direct evidence for acceleration, but they were accepted quickly in part because studies of large scale structure and CMB anisotropies already argued indirectly in favor of a cosmological constant (e.g., [3,4,5]). The supernova evidence has become much stronger on its own terms [6], and measurements of the first acoustic peak in the CMB power spectrum [7,8,9] greatly strengthen the overall case for dark energy by showing that the *total* energy density of the universe is close to critical. This rules out the nearly empty open models that might otherwise be marginally compatible with the supernova data, and in combination with dynamical evidence for a low density of clustered matter ($\Omega_m \sim 0.2 - 0.4$), it demonstrates that there must be some additional entry in the cosmic energy budget. Unless this new component has negative pressure, the inferred ages of globular cluster stars conflict with the most convincing recent estimates of the Hubble constant [10]. We thus have three largely independent lines of argument for dark energy: the supernova Hubble diagram, the dynamical evidence for low matter density, and the age of the universe. In addition, we have the astonishing success of the inflationary cold dark matter paradigm in explaining high-precision measurements of CMB anisotropy, galaxy clustering, the Lyα forest, gravitational lensing, and other phenomena (see [11] for an up-to-date discussion), a success that vanishes if there is no dark energy component.

From a theoretical point of view, there are three different aspects of the dark energy puzzle. The first is the "old" cosmological constant problem: a naive application of quantum field theory suggests that the energy density associated with vacuum zero-point fluctuations should be of order one Planck mass per cubic Planck length, which exceeds the observational bound by ~ 120 orders of magnitude. Since the only natural number that is $\sim 10^{-120}$ is zero, it is usually assumed that a correct calculation will someday show that the true value of the fundamental vacuum energy is exactly zero or vanishingly small. This leaves us with the second problem: what *is* causing the universe to accelerate? Finally, there is the coincidence problem: why is the dark energy density comparable to the matter density today, when most models predict that they were very different in the past and will be very different in the future?

Faced with these conundra, theorists have proposed an impressive variety of

possible solutions. The most common class introduces a new energy component, typically a scalar field, and associates dark energy with the the field's potential energy or kinetic degrees of freedom. These solutions do not address the "old" cosmological constant problem, so some other mechanism must set the fundamental vacuum energy to zero. Alternatively, the dark energy really could be the fundamental vacuum energy, and it has its remarkably small value as a consequence of not-yet-understood aspects of quantum gravity, or because it varies widely throughout an inflationary "multiverse" and anthropic selection restricts life to regions where it is small enough to allow structure formation [12,13,14], or because gravitational back reaction causes it to oscillate in time producing alternating phases of acceleration and deceleration [15]. Finally, there is the possibility that GR itself must be modified, perhaps pointing the way to extra spatial dimensions or testable consequences of string theory. No matter what, the existence of cosmic acceleration implies a fundamental revision to our understanding of the cosmic energy contents, or particle physics, or gravity, or all of the above. The combination of theoretical importance and observational difficulty make the dark energy problem a worthy science driver for an ambitious wide field imaging mission in space.

2   Constraining Dark Energy

If dark energy is not a simple cosmological constant, then it is generically expected to have spatial inhomogeneities that affect CMB anisotropies and large scale structure. However, unless we are lucky (i.e., dark energy properties are just what is required to produce large inhomogeneities), these effects are too weak to detect in the face of cosmic variance. If cosmic acceleration is a consequence of modified gravity, then laboratory measurements or tests for intermediate-range forces could also turn up clues. But again, we would have to be lucky.

The only generic method of studying dark energy is to measure its influence on cosmic expansion history, described by the Friedmann equation

$$H(z) = H_0 \left[\Omega_m (1+z)^3 + \Omega_k (1+z)^2 + \Omega_\phi \frac{\rho_\phi(z)}{\rho_{\phi,0}} \right]^{1/2}. \tag{1}$$

Here the Ω_x refer to densities at the present day ($z = 0$) in units of the critical density, and $\Omega_k \equiv 1 - \Omega_m - \Omega_\phi$. I have ignored the radiation term $\Omega_r(1+z)^4$, which becomes important only at high redshifts. A "fluid" with equation of state $p = w\rho c^2$ has $\rho_\phi(z) = \rho_{\phi,0}(1+z)^{3(1+w)}$, so a true cosmological constant with $\rho_\phi(z) = \rho_{\phi,0}$ has $w = -1$. The effects of modified gravity solutions that change the Friedmann equation itself may be well approximated by a dark

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energy model with some $\rho_\phi(z)$, but perhaps not in all cases.

There have been many proposed tests of dark energy based on supernovae, radio galaxies, the virial masses, X-ray properties, or Sunyaev-Zel'dovich decrements of galaxy clusters, large scale structure traced by galaxies, clusters, or the Lyα forest, galaxy ages, or various aspects of strong or weak gravitational lensing (see [16] for an extensive but not exhaustive list). Nearly all of these tests depend on some combination of four fundamental observables: the Hubble parameter $H(z)$, the distance-redshift relation $d(z)$, the age-redshift relation $t(z)$, or the linear growth factor of mass fluctuations $D_1(z)$. The evolution of $H(z)$ is governed by the Friedmann equation (1). The comoving line-of-sight distance to an object at redshift z is

$$d(z) = \frac{c}{H_0} \int_0^z dz' \frac{H_0}{H(z')} \ . \tag{2}$$

In a flat ($\Omega_k = 0$) universe, the angular diameter and luminosity distances measured with standard rulers or candles are related to $d(z)$ by cosmology-independent powers of $(1 + z)$. In a curved universe, they are proportional instead to the transverse comoving distance,

$$d_M(z) = \frac{c}{H_0} |\Omega_k|^{-1/2} S_k \left(|\Omega_k|^{1/2} \frac{H_0}{c} d(z) \right) \ , \tag{3}$$

where $S_k(x) = \sin x$ or $\sinh x$ for $k = +1, -1$ [17]. The age of the universe at redshift z is

$$t(z) = \int_z^\infty \frac{dz'}{(1 + z')H(z')} \ . \tag{4}$$

The linear growth factor is the solution to the differential equation

$$\ddot{D}_1 + 2H(z)\dot{D}_1 - \frac{3}{2}\Omega_m H_0^2 (1 + z)^3 D_1 = 0 \ . \tag{5}$$

The solution can be written as a simple integral only for special values of w (including $w = -1$), but one can gain intuition using the approximation $d \log D_1 / d \log a \equiv f(\Omega_m) \approx \Omega_m^{4/7}$, implying

$$\log D_1(z) = -\int_0^z \frac{dz}{1 + z} [\Omega_m(z)]^{4/7} \ , \tag{6}$$

where $\Omega_m(z) = \Omega_m(1 + z)^3 / [\Omega_m(1 + z)^3 + \Omega_k(1 + z)^2 + \Omega_\phi \rho_\phi(z)/\rho_{\phi,0}]$ is the
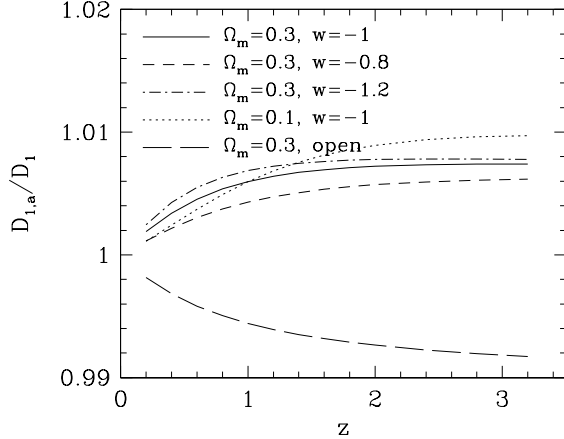
Fig. 1. Ratio of the approximate growth factor $D_1(z)$ calculated by equation (6) to the exact value for various combinations of Ω_m and w, including an open universe model with $\rho_\phi = 0$.

matter density parameter at redshift z. Here I have normalized $D_1(z = 0) \equiv 1$ and used the index 4/7 that is accurate for $\Omega_m(z) \approx 1$ [18] instead of the value 0.6 that is more accurate for low Ω_m [19]. Figure 1 shows that the approximation (6) is accurate to better than 1% for a wide range of models; the accuracy degrades to $\sim 3\%$ using $f(\Omega_m) = \Omega_m^{0.6}$.

The virtue of discussing dark energy tests in terms of these observables is that one can see whether and how different observational strategies complement each other. For example, Type Ia supernovae and other standard candle or standard ruler methods measure $d(z)$ directly, while the abundance of clusters as a function of redshift depends on both $D_1(z)$ and the volume element $dV \propto d^2(z)/H(z)$. Weak lensing observables depend on $D_1(z)$, $d(z)$, and $H(z)$, depending on just what properties (power spectrum, skewness, etc.) are measured. The Alcock-Pacyznski [20] anisotropy test measures the product $d(z)H(z)$.

Current observational evidence demonstrates that $\rho_\phi \neq 0$, and it is compatible with a model having $\Omega_m = 0.3$, $\Omega_\phi = 0.7$, $\Omega_k = 0$, and $w = -1$. To show that dark energy is not a cosmological constant, one must show that the data are incompatible with constant $\rho_\phi(z)$. To show that the equation of state parameter w is time-variable, one must show that the data are incompatible with $\rho_\phi(z) \propto (1+z)^n$ for any choice of n. Let us begin with the first, much less demanding problem, and make the further simplifying assumption that $\Omega_k = 0$. While CMB measurements provide a precise value of the angular diameter distance to the surface of last scattering, they do not demonstrate that $\Omega_k = 0$ at the level of precision of interest here (see, e.g., [21]), so exact spatial flatness is a theoretically motivated assumption rather than an observational constraint.
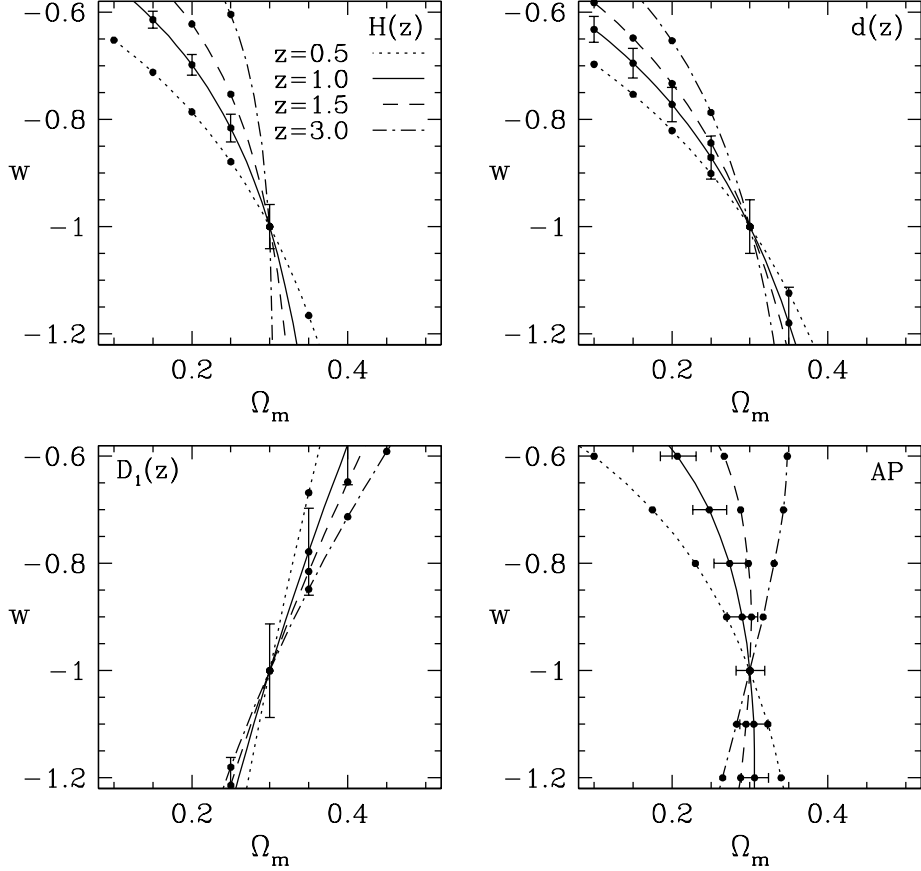
Fig. 2. Contours in the $\Omega_m - w$ plane along which the values of expansion history observables are constant, assuming $\Omega_k = 0$, constant w, and fixed H_0. The four line types correspond to four redshifts, as indicated. Error bars show the uncertainty in w or Ω_m associated with a $\pm 1\%$ uncertainty in the observable, at $z = 1$.

With these assumptions, the parameters to be constrained are w and $\Omega_m = 1 - \Omega_\phi$. The decelerating effect of matter is proportional to Ω_m, and the accelerating effect of dark energy is proportional to Ω_ϕ but also depends on the pressure-to-density ratio w. For any given redshift and observable, there is a contour in the $\Omega_m - w$ plane along which the value of the observable is constant. Figure 2 shows such contours at $z = 0.5$, 1, 1.5, and 3 for the Hubble parameter $H(z)$, the distance $d(z)$, the linear growth factor $D_1(z)$, and the Alcock-Pacyznski (AP) parameter $H(z)d(z)$, assuming a fixed value of H_0. Error bars on the $z = 1$ curves show the impact of a 1% error on the corresponding observable. I have not included plots for $t(z)$ because I think it unlikely that the systematic errors on absolute age measurements can be brought low enough to teach us anything we do not already know about dark energy.

There are several lessons to be drawn from Figure 1. First, interesting ($\Delta w \sim 0.1$) constraints on w typically require $\sim 1 - 2\%$ measurements at a given redshift, if Ω_m is known independently. Second, measurements of different

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observables, or of the same observable at different redshifts, can break the degeneracy between Ω_m and w. Combinations of $D_1(z)$ with $d(z)$ or $H(z)$ are especially useful in this regard, because Ω_m changes have opposite effects. Applications of the AP test at $z \sim 2-3$, which may be possible with the Lyα forest [22,23,24], would yield Ω_m constraints that are nearly independent of w, also helping to break degeneracies. The AP test is independent of uncertainties in H_0, as are measurements of $d(z)$ that use candles or rulers calibrated in the local Hubble flow and measurements of $H(z)$ that compare a velocity scale measured at high redshift to a lengthscale measured from local galaxy redshift surveys. However, the CMB provides a standard ruler in absolute Mpc, so tests that use this standard ruler also require a precise determination of H_0 (see [25] for an excellent discussion of this point).

Constraining w becomes substantially more difficult if we drop the assumption that $\Omega_k = 0$, since Ω_m and Ω_ϕ can now vary independently. Most error forecasting papers do not investigate this case in detail, and I have not done so myself. Curvature affects all observables through the $\Omega_k(1+z)^2$ term in the Friedmann equation, and it additionally affects the distance-redshift relation (and thus also the AP parameter) through geometrical effects. Thus, complementary studies using distance/AP measurements and growth factor/$H(z)$ measurements would be especially valuable for this case, as would precise independent measurements of Ω_m from large scale structure. The CMB determines the combination $\Omega_m h^2$ very well, but a tight CMB constraint on Ω_m again requires a precise measurement of H_0.

Demonstrating time-variation of w is in general very difficult. If the effective value of w is close to -1 today, as the observations imply, then the ratio of the matter density to the dark energy density scales as $\sim (1+z)^3$. Dark energy is therefore dynamically unimportant even at moderate redshift ($z \sim 2$), leaving little leverage to show that the scaling of $\rho_\phi(z)$ is *not* adequately described by a power law $(1+z)^n$ as expected for constant w. While quintessence models generically predict that w varies in time, the effects of that variation are much too weak to detect in typical cases [16].

Figure 3 illustrates examples from a class of models in which

$$\rho_\phi(z) = \rho_{\phi,0}(1+z)^n \left[1 + \left(\frac{1+z}{1+z_c}\right)^\alpha\right]^{(m-n)/\alpha}, \tag{7}$$

so that $\rho_\phi \propto (1+z)^n$ at $z \ll z_c$ and $\rho_\phi \propto (1+z)^m$ at $z \gg z_c$. Kujat et al. [16] considered a similar class of models, but the additional parameter α allows the transition in scaling behavior at z_c to be arbitrarily sharp, instead of necessarily occurring over an interval $\Delta z \sim z_c$. Long-dashed lines show a model that I consider maximally optimistic from the point of view of detecting time variations of w: the energy scaling changes from $\rho_\phi \propto (1+z)^3$ to $\rho_\phi = $ constant
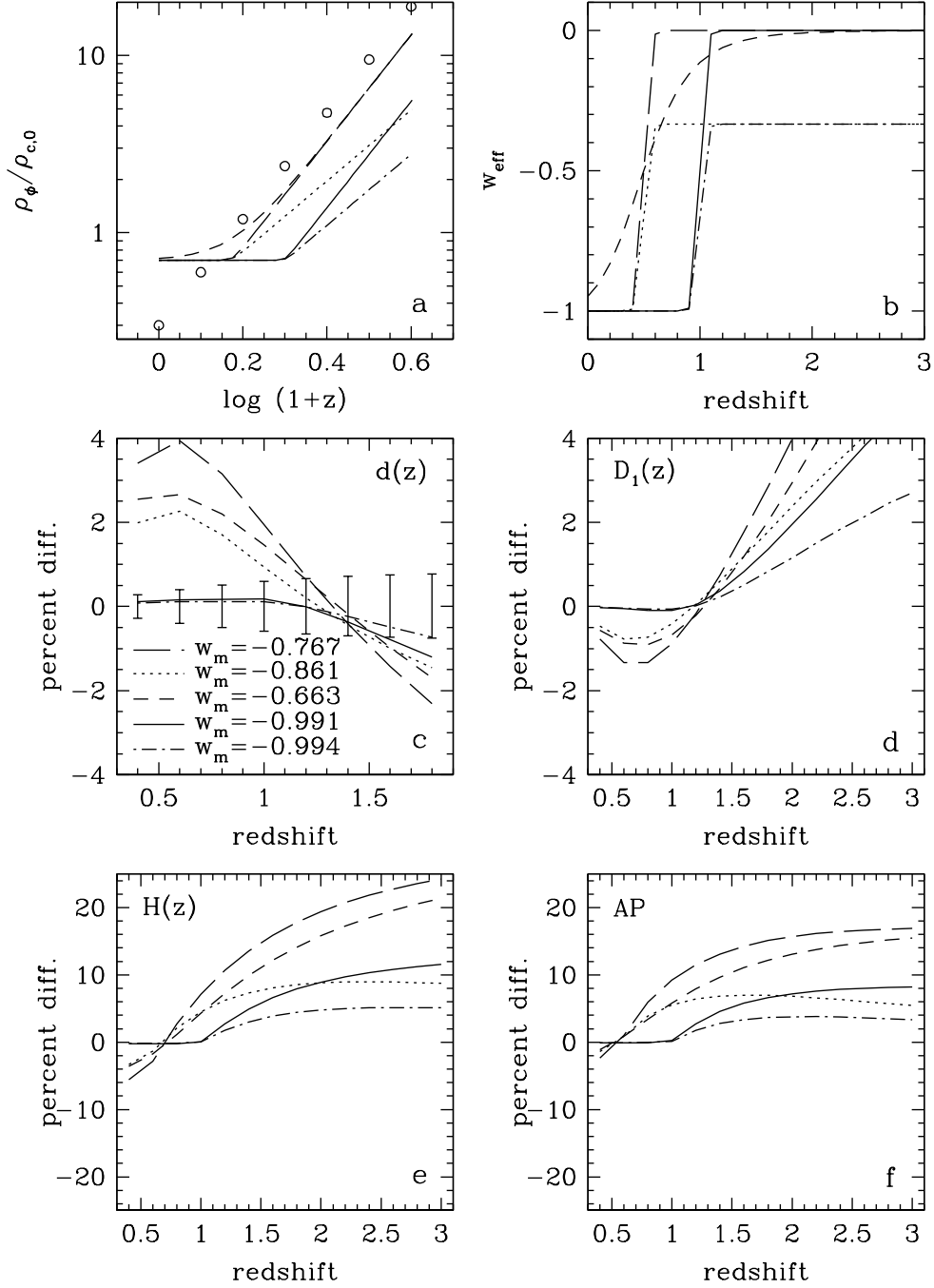
Fig. 3. Time-varying w models, with the parameterization of equation (7). Panels a and b show the evolution of $\rho_\phi(z)$ and $w_{\text{eff}} \equiv \frac{1}{3} d \log \rho_\phi / d \log(1 + z) - 1$ for five different parameter combinations. Circles in panel a show the matter density. Panel c plots the percentage difference between the time-variable model and the constant-w model that best matches its distance predictions over the range $0.4 \le z \le 1.8$. For reference, error bars show the percentage difference caused by changing Ω_m by ± 0.01. Panels d, e, f show percentage differences in other observables relative to the same constant-w models. Note changes in horizontal and vertical axis scales.

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at $z_c = 0.5$, and the transition is nearly instantaneous. One can argue that $m = 3$ is a "natural" (or at least interesting) choice because a model in which ρ_ϕ scales like the matter density during the matter dominated era has a better chance of solving the coincidence problem [26,27]. However, the low redshift and sharpness of the transition make it as easy as possible to detect, and there is no obvious reason that nature should be so kind to us. The short-dashed lines show a case with a smooth transition at $z_c = 0.5$, and solid lines show a sharp transition at $z_c = 1$. Dotted and dot-dashed lines show sharp transitions at $z_c = 0.5$ and 1, but with high-redshift energy scaling $\rho_\phi \propto (1 + z)^2$ instead of $(1 + z)^3$.

Figure 3c shows the key result, focusing on the redshift range $z = 0.4 - 1.8$ where a space-based Type Ia supernova survey is likely to produce its tightest constraints. For each of the time-variable models, I have identified the constant-w model that most nearly matches its $d(z)$ relation, minimizing the summed absolute fractional differences over the range $0.4 \leq z \leq 1.8$. The quantity plotted is the percentage difference of $d(z)$ between the time-variable model and the "matched" constant-w model.

For the maximally optimistic model, the prospects look quite good. There is a 6% variation in distance between this model and the matched constant-w model, and a mission like SNAP or DESTINY could reasonably expect to measure such a variation at high significance. For a smooth transition (short-dashed line) or high-redshift scaling $m = 2$ (dotted line), the variation is $\sim 3\%$, still plausibly within reach. However, if the transition redshift is $z_c = 1$, then the maximum difference in distance is $\leq 1\%$, and this is reached only in the highest redshift bin.

Following Figure 2, one might hope that measurements of the growth factor would complement distance measurements, yielding combined tests of time variation of w that are much more powerful than either measure alone. Unfortunately, the complementarity in the $\Omega_m - w$ space does not translate to complementarity for this new problem, at least at $z < 2$. Figure 3d shows percentage differences in $D_1(z)$ relative to the *same* constant-w models that best match the distance measurements. [I have been lazy and used the approximation (6).] The differences in $D_1(z)$ are smaller than those in $d(z)$, even though the values of w were chosen to match the latter not the former. The discriminatory power of $D_1(z)$ does increase at higher redshifts, especially for $m = 3$, because in this case ρ_ϕ freezes in at a constant fraction of ρ_m (Fig. 3a), so $d\log D_1/d\log(1 + z) \approx \Omega_m^{4/7}$ is always depressed below unity. The Lyα forest might eventually allow a measurement of D_1 at $z \sim 3$ with enough precision to be interesting in this regard [28,29]. The comparison of CMB anisotropy amplitude to the matter power spectrum amplitude at low redshift would be more powerful still, if the latter can be measured precisely enough and the effects of dark energy variation can be separated from those of the reioniza-

tion optical depth and tensor fluctuations, both of which affect the relative amplitudes of CMB and matter power spectra.

The high-redshift behavior of $\rho_\phi(z)$ has a stronger impact on $H(z)$ because there is no integrated contribution from low redshift like those in equations (2) and (6). Thus observations that directly constrain $H(z)$ at $z > z_c$ can add discriminatory power even if they are much less precise than the distance measurements at $z < 2$. Because the $H(z)$ variations are much larger than the $d(z)$ variations, the AP test effectively serves the same role as an $H(z)$ measurement at high redshift. The Lyα forest and redshift surveys of Lyman-break galaxies both offer possible routes to inferring these quantities at the few percent level [30,22,23,24,31,32,33], though it is not yet clear that systematic errors in these approaches can indeed be controlled to this level of accuracy.

The experiment in Figure 3 is similar to that carried out by [16]. If my conclusions here seem a bit more optimistic, it is mainly because I have zeroed in on the subset of models for which time variation of w is easiest to detect, and partly because the prospect of a wide-field imager in space makes the idea of percent-level distance measurements seem worthy of serious discussion, not simply pie in the sky. Note also that Figure 3 is restricted to flat models. Evidence for strong time-variation of w would be surprising enough that one would want to consider observationally allowed values of $\Omega_k \neq 0$, thus giving more freedom to reproduce the data with a constant-w model.

3 Considerations for a Space-Based Dark Energy Experiment

The discovery of dark energy has profound implications for cosmology and particle physics, even if we do not yet understand what all of those implications are. A demonstration that the dark energy density has changed over time (i.e., that $w \neq -1$) would be an achievement of comparable magnitude. A precise measurement of w would rule out many models of dark energy and provide a clear target for predictive models that might emerge from fundamental physics. A demonstration that the effective value of w has changed with time would be much more remarkable still, eliminating most models and providing a strong clue to the physics of dark energy.

While $w = -1$ has history on its side, it has no particular claim to physical plausibility in the absence of a successful theory of fundamental vacuum energy. I thus see no reason to conclude that w is likely to be -1 just because current data are consistent with this value, and improved observations that restricted it to, say, $w = -1.0 \pm 0.1$ would not change this situation. If $|w - 1|$ is ~ 0.15 or more, then there are good prospects for detecting this departure with the few percent level of precision that can be obtained by ground-based

experiments, especially as different approaches can provide complementary constraints and independent tests (Fig. 2). However, a demonstration that $w = -0.95$ would be just as profound as a demonstration that $w = -0.85$, and a demonstration that $w = -1.05$ would be still more striking, implying physics even weirder than that we are currently forced to accept. The level of precision and control of systematics required to detect such small departures from $w = -1$ can only be achieved from space.

Detecting time-variation of w would almost certainly require a space-based investigation, since the complementarity of different methods is largely lost, and the measurements must probe the largest possible redshift range to achieve maximum leverage. Even with the capabilities of a mission like SNAP or DESTINY, I think one must deem a convincing detection of time-variation unlikely, because for slow variations there will always be a constant-w model that produces nearly identical results over the redshift range where dark energy is dynamically important. There is one class of "generic" models in which time variation is *relatively* easy to detect: dark energy scales like matter during the matter-dominated era and transitions to a constant density when Ω_m falls significantly below one. However, even in this class the transition must occur quickly and at low redshift to be observable.

Because I consider a detectable value of $|w - 1|$ to be *a priori* much more plausible than a detectable level of time-variation, I think that one should not sacrifice precision on the former to gain leverage on the latter. If one assumes that supernova distance measurements in redshift bins at $z \leq 1$ will be limited by systematic uncertainty rather than by statistics, then there is no reason not to pursue higher redshift objects in the quest for time variability. This is the attitude of the SNAP team, and it seems to me entirely reasonable. However, if cost constraints force consideration of a descoped mission, then the choice between precision on w and leverage on time-variation may become unavoidable.

The very fact that a space-based dark energy mission will be much more capable than any ground-based experiment raises the stakes, since any discovery made by this mission would be unrepeatable for many years to come. Everyone will have slightly different views on what statistical confidence level is needed for an "interesting" result on dark energy. In my view, satisfying evidence for $w \neq -1$ would require one result with $> 3\sigma$ significance or two consistent results with $> 2\sigma$ significance, along with convincing tests for systematic effects. Time-variation of w demands a higher standard because of its lower prior probability, maybe one $> 4\sigma$ result or two $> 3\sigma$.

To my mind, the great advantage of the SNAP concept for a dark energy mission relative to the DESTINY concept is that the Type Ia supernova experiment and the weak lensing experiment look capable of achieving similar

statistical precision, with largely independent systematic uncertainties. With DESTINY, there is no obvious backup method that comes close to the precision of the supernova search, making it harder to show that an unexpected result on dark energy is actually correct. The comparison is not entirely fair, since DESTINY would likely be much less expensive than SNAP, and it is no surprise that it has narrower scope. Either of these missions would be a great step forward in our efforts to unravel the most intriguing cosmic mystery of our time, and, as we heard throughout this workshop, either would enable a vast array of other fascinating investigations.

Acknowledgements

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